

Millisecond Pulsar Searches and Double Neutron Star Binaries

J. Middleditch

*Modeling, Algorithms, and Informatics, MS B265, Computer & Computational
Science Division, Los Alamos National Laboratory, Los Alamos, NM 87545*

jon@lanl.gov

ABSTRACT

A unified strategy is developed that can be used to search for millisecond pulsars (MSPs) with \sim solar mass companions (including neutron star companions in double neutron star binaries [DNSBs]) belonging to both very short period binaries, and those with periods so long that they could be appropriate targets for acceleration searches, and to bridge the gap between these two extremes. In all cases, the orbits are assumed to be circular. Applications to searches for binary pulsars similar to PSR J0737-3039 are discussed. The most likely candidates for more DNSBs consist of weakly magnetized neutron stars, spinning only moderately fast, like J0737-3039A, with periods generally longer than 15 ms, though this issue is not yet settled. Because of the similarity between the MSP components of DNSBs, and the longer period MSP population specific to massive condensed or core collapsed globular clusters, as well as the uncertainties about accretion-driven spinup, doubts linger about the standard models of DNSB formation.

Subject headings: binaries: close — pulsars: general — pulsars: individual (J0737-3039) — stars: neutron — supernovae: general

1. Introduction

The discovery of the double binary pulsar system PSR J0737-3039A,B (hereafter J0737 – Burgay et al. 2003; Lyne et al. 2004) has rekindled interest in computation intensive searches for other similar systems, because their coalescence events will be detectable as sources to gravitational observatories (Kalogera et al. 2004), that will most likely be stronger than merger-induced core collapse supernovae (SNe) if very much less common (Middleditch 2004a, hereafter M04a). None of the other seven double neutron star binaries (DNSBs) have detected pulsar companions. Four of these (B1534+12, J1756-2251, B1913+16, and M15C)

have orbital periods of 10, and 3*8 hours respectively (Wolszczan 1991; Faulkner et al. 2004; Hulse & Taylor 1975; Prince et al. 1991) one has a period of 28 hours (J1829+2456, Champion et al. 2004) and the remaining two (J1518+4904 and J1811-1736) have periods of 9 and 19 days (Nice, Sayer, and Taylor 1996; Lyne et al. 2000). In addition, three (J1811-1736, B1913+16, and M15C) have eccentricities of 0.83, 0.62, and 0.68, while four (J1518+4904, B1534+12, J1756-2251, and J1829+2456) have eccentricities of 0.25, 0.27, 0.18, and 0.14. By contrast, the eccentricity for J0737, with an orbital period of only 2.4 hours, is only 0.088. Should this trend continue as DNSBs evolve, then deeper, brute force searches using only circular orbits and quantum computers if necessary, may ultimately find more (however, the lifetime of such close binary systems is very short). Other applications include millisecond pulsar (MSP) searches in low mass X-ray binaries (e.g., Dib et al. 2004), and very young supernova (SN) remnants (SNRs) in which a newly-born pulsar is spinning down rapidly and/or mimicking binary motion through precession (Nelson, Finn, & Wasserman 1990; Middleditch et al. 2000, hereafter M00).

Even membership in a binary system with circular orbits, however, can drastically reduce the detectability of an MSP to conventional search techniques. In fact, if J0737A&B were moderately weaker at 1,400 MHz than 1.6 and 1.3 mJy, they would never have been detected even by acceleration searches. In addition, through the use of radio imaging techniques at sites such as the VLA, and others in Australia, point sources will be mapped that will be pulsar candidates, but will also be near the threshold of detection as pulsars even by conventional acceleration searches. Thus, many such mapped sources will also require brute force binary pulsar searches to uncover their pulsar parameters.

The next section (§2) begins with a discussion of acceleration searches (§2.1), continues with a discussion of short period binary searches (§2.2), and derives how to transition between the two search extremes. Section 2.3 gives an example of a short period binary search, and §2.4 discusses ways to maintain the effectiveness of such searches with much less required computation. The following sections (§3 and §4) discuss how the DNSBs could affect MSP searches, and conclude.

2. Technique

2.1. Acceleration Searches

The problem to be solved by future searches, then, is how to compensate, *efficiently, but still completely*, for the possibility that a target being searched may be a pulsar member of a short period binary system, and how to extend such searches into the range of longer

binary periods. By “short” we mean a binary period that is short enough so that the drift in pulse frequency due to the pulsar’s acceleration in the binary system can be greater than one or two wavenumbers of frequency (cycles (observation time)^{−1}). Expressed in terms of the physical parameters of the binary system, this constraint becomes:

$$f'_{max}T^2 = 1.844 * 10^{-5}(f/650)T^2P_d^{-4/3} \sin i (M/2.4M_{\odot})^{1/3} 2.4M_c/M \geq 1 , \quad (1)$$

where f is the pulsar repetition frequency in Hz, f'_{max} is the maximum absolute value of the time derivative of the pulse frequency caused by the orbital motion, T is the time duration of the observation of the pulsar, P_d is the orbital period in days, $\sin i$ is the sin of the orbital inclination, i , and M is the total mass of the binary system, i.e., the mass of the pulsar, M_p , plus the mass of the companion star, M_c . Equation (1) shows that even an observation as short as one(five) minute(s) for a 0.1/1.0 day binary period, can be affected for a 650 Hz pulsar orbiting a solar mass companion.

Thus we need to know how densely we must sample the parameters of the binary orbit in order that we lose, in the worst case situation, only 10%, 20%, or 40% of the power that a pulsar would produce in a Fourier transform of a given observation, relative to the power produced *if* the orbital parameters of our data-taking apparatus (or of our time series-stretching software) were *perfectly* matched to the orbital parameters of the pulsar. In order to solve this problem, it is first easier to investigate the problem in the limit of the “longer” short orbital periods, where we need only concern ourselves with matching the first two time derivatives of frequency, f' and f'' .

The errors that can be associated with the measurement of the rate of frequency drift in time, f' , and the next time derivative, f'' , are given by:

$$\delta f' = \sqrt{90}/(\pi\sqrt{P_F}T^2) , \quad (2)$$

$$\delta f'' = 6\sqrt{105}/(\pi\sqrt{P_F}T^3) , \quad (3)$$

where P_F is the Fourier power produced by the periodic pulsations in a power spectrum normalized so that the local mean power near (but not at) the pulsar frequency (or harmonic being discussed) is unity. These errors apply only when f' and f'' are to be measured while the remaining parameters, f and f'' , or f and f' respectively are known a priori to the measurement. The derivation of the formulae for δf (which is given by $\delta f = 3/(\pi\sqrt{6P_F}T)$) and $\delta f'$ are outlined elsewhere (Ransom, Eikenberry, & Middleditch 2002, hereafter REM02); the formula for $\delta f''$ can be derived with similar arguments. (The formula for $\delta f'''$ can not be so derived, possibly because pure f''' is almost indistinguishable from an ordinary periodic frequency modulation, which produces little effect at the central frequency. However the 2nd harmonic of the 2.14 ms pulsations from SN1987A during 31 Oct. ‘95 was so highly phase

modulated that the Fourier peak at the central frequency was starting to *split* – M00.) Using a continuous Fourier interpolation and the first, second, and third derivatives of the Fourier amplitudes wrt the frequency (f), f' , and f'' can be estimated for a small range about 0, from the Fourier amplitudes close to f (Middleditch & Cordova 1982; Middleditch, Deich, & Kulkarni 1992, hereafter MDK).

Since a power spectrum with unit mean has the statistical properties of one half of the χ^2 for two degrees of freedom, we could have defined the errors given in equations (2) and (3) as the deviations of the parameters f' and f'' (taken one at a time) which reduced the power of a Fourier peak by 0.5 units below the maximum value. Thus we can rework equations (2) and (3) to solve for the deviations which reduce the peak power by 10% (i.e., reduced to 0.9 of the peak value):

$$T^2 \delta f'_{90\%} = 3\sqrt{2}/\pi , \quad (4)$$

$$T^3 \delta f''_{90\%} = 6\sqrt{21}/\pi . \quad (5)$$

By differentiation of the Doppler equation for f , we can relate f' and f'' to a function of the orbital phase, Ωt :

$$f' = \frac{f\Omega^2 a_p \sin i}{c} \cos(\Omega t) , \quad (6)$$

$$f'' = \frac{f\Omega^3 a_p \sin i}{c} \sin(\Omega t) , \quad (7)$$

where t is time, c is the velocity of light, $\Omega = 2\pi/P$, where P is the orbital period, and a_p is the size of the orbit of the pulsar in the binary system:

$$a_p = 1.633 * 10^{11} P_d^{2/3} \left(\frac{M}{2.4M_\odot} \right)^{1/3} * 2.4M_c/M \text{ (cm)} . \quad (8)$$

We can see from equations (6) and (7) that f' and f'' represent orthogonal coordinates of the orbital system (ignoring $\sin i$ for the moment), with f' measuring the depth into the binary system along the projected line of sight (conjunction axis – Middleditch & Nelson 1976), and f'' measuring the extent of the binary system in the plane of the sky, perpendicular to the conjunction axis (i.e., along the line of quadratures).

Thus, if we wanted to search our data for a pulsar in a not-too-short orbital period system, we would superpose a rectangular lattice of points over a circular orbit, where the lattice spacing is given by twice the tolerances of equations (4) and (5). At quadrature, with both binary members at maximum elongation (and thus in the plane of the sky, or $\Omega t = \frac{\pi}{2}$ or $\frac{3\pi}{2}$), f' is near 0, but changing very rapidly, and hence f'' is near one of its two extremes, but changing only very slowly. Here also the *differentials* of f' and f'' resolve into

relations that depend only upon the differentials of one of the two binary parameters, a_p and orbital phase, $\Phi = \Omega t$:

$$|\delta f'|_{max} = \frac{\delta \Phi f \Omega^2 a_p \sin i}{2c} ; \left(\Phi = \frac{\pi}{2}, \frac{3\pi}{2} \right), \quad (9)$$

$$|\delta f''|_{max} = \frac{\delta a_p \sin i f \Omega^3}{2c}; \quad \text{,,} \quad . \quad (10)$$

Using equations (4) and (5) for the tolerances, we can derive an expression for the tolerances of the binary parameters, a_p , and Φ :

$$\delta \Phi = \frac{6\sqrt{2}c}{\pi f (\Omega T)^2 a_p \sin i} \quad (radians) , \quad (11)$$

$$\delta a_p = \frac{12\sqrt{2}c}{\pi f (\Omega T)^3 \sin i}; \quad \left(\Phi = \frac{\pi}{2}, \frac{3\pi}{2} \right) . \quad (12)$$

The above equations give the mismatch of the orbital parameters that would produce a 10% loss of the power of a Fourier peak (per parameter).

The simplicity of equations (11) and (12) has provided part of the motivation behind the quadrature search technique. This involves scanning through a data set large enough to contain at least one quadrature, plus at least half of the analysis size length, $T/2$, and tracking the data through the orbital motion, with increments in orbital parameters close to those given by equations (11) and (12), hoping to detect the pulsar. The other part of the motivation for the quadrature search has been due, in part, to computing limitations, either because of an upper limit to the size of the Fourier transforms that could be done, or an upper limit to the total amount of computing time available. Here, however, we wish to develop a search strategy that can make use of the entire set of data taken.

Since the rectangular lattice of f' and f'' values has exactly the same density over the entire orbital plane, the experimenter could do just as well by searching the data for Doppler-shifted pulsar behavior at **any** a priori selected orbital phase domain, including conjunction. In this case, small steps must be taken in a_p , but large steps are allowed in Φ . The only danger is that second order deviation of f' from the maximum value caused by the now cruder steps in Φ (set by the tolerance on f''), could exceed the tolerance set by the finer steps in a_p . With a little calculation, one can show that the ratio of the step size in Φ required by f'' to that required by the 2nd order change in f' near maximum, can be given as:

$$\frac{\delta \Phi_{f''}}{\delta \Phi_{f', 2nd}}(\Phi = 0, \pi) = 0.00223 \left(\frac{650}{f} \right)^{1/2} \left(\frac{P}{T} \right)^2 P_d^{-1/3} \left(\frac{2.4 M_\odot}{M} \right)^{1/6} \left(\frac{M}{2.4 M_c} \right)^{1/2} (\sin i)^{-1/2} . \quad (13)$$

Thus, paradoxically, the conjunction searches for millisecond pulsars are in trouble due to second order changes in f' until the orbital period, P , becomes *shorter* than ~ 21 times the observation time, T , unless f is *faster* than 650 Hz, or the companion mass is large. The rationale behind this statement is that the longer orbital periods allow much *larger* values of $\delta\Phi$ at conjunction before f'' is a problem, i.e., the dependence of P on $\delta f''$ is stronger and dominates the two effects. Eventually, however, as P becomes much shorter, the approximation involved in equating f' and f'' to the orbital parameters has broken down due to the neglect of the higher derivatives.

Before beginning our discussion of short period binaries, we want to find out how to transition between the two extremes of searches. We can do this by focusing our attention on the tolerance in $\delta\Phi * a_p$ ($\Phi = \frac{\pi}{2}, \frac{3\pi}{2}$) given by equation (11). In the limit of $\Omega T \leq \sqrt{42}$, or basically for all observations of duration *less* than one orbital period, this tolerance is always more stringent than that given for δa_p in equation (12). It is identical to the tolerance on δa_p dictated by the tolerance of f' at the conjunctions ($\Phi = 0, \pi$), when f' has its extreme values that multiplication by $\delta a_p / a_p$ can lead to (potentially large) fractional variations. The point we wish to make here, then, is that *no parameter spacing needs to be finer than this tolerance as the orbital period, P , gets long in comparison to the data stream length to be analyzed, T .*

2.2. Searches in Short Period Binaries

Now that the asymptotic value for the spacing of the orbital parameter space has been found, we can investigate the situation for the very short period binary systems, with approximately one or more binary orbits occurring during the data interval, T , with the goal of smoothly connecting pulsar searches with short and longer orbital periods. In the situation where one or more orbits occur during the run time, T , a binary orbit causes a phase modulation of amplitude:

$$Z_\phi = \frac{2\pi f a_p \sin i}{c} \quad (\text{radians}) , \quad (14)$$

so that the equation of the data becomes:

$$g(t) = a * \text{noise} + b * (1 + \cos(2\pi f t + \phi_0 + Z_\phi * \cos(\Omega t + \Phi_0))) . \quad (15)$$

The Fourier transform of this signal will produce peaks in the Fourier power spectrum at f , $f \pm \frac{\Omega}{2\pi}$, $f \pm 2\frac{\Omega}{2\pi}$, \dots , $f \pm n\frac{\Omega}{2\pi}$, \dots , each with complex Fourier amplitude proportional to $(\sqrt{-1})^n J_n(Z_\phi)$, where the J_n are Bessel functions of n^{th} order, and a technique has been developed that exploits this pattern in the Fourier power spectrum (Ransom, Cordes, and Eikenberry 2003). In this brute force method, however, the time series is stretched to match

a particular set of orbital parameters and the mismatch of these to the true set generates a particular error vector, δZ_ϕ in the complex Z_ϕ plane, constructed, as other complex planes, to track the modulus and phase of such vectors. A residual phase modulation then ensues because the error vector has a finite length, and can also migrate during the observation time due to orbital period mismatch. For the sake of simplicity, we will assume for now that the phase modulation has a constant amplitude and phase. Power is lost to this phase modulation from the main peak, whose height is proportional to:

$$J_0^2(\delta Z_\phi) \approx 1 - (\delta Z_\phi)^2/2 . \quad (16)$$

If we are completely ignorant of the orbital phase and projected orbital radius of a particular binary system, then we must search over the Z_ϕ plane with tile sizes which can be determined from equation (16), i.e.,

$$\delta Z_{\phi,90\%} = 1/\sqrt{5} \quad (radians) . \quad (17)$$

Thus for a worst case mismatch of orbital phase modulation of $1/\sqrt{5}$ radians, the worst case loss of Fourier power will be only 10%.

To cover the Z_ϕ plane efficiently, we choose a hexagonal tiling with a side dimension of $1/\sqrt{5}$. The hexagonal tile has the same area ($3\sqrt{3}/10$) as a square tile of 0.721 radians on a side. For a tile on the plane with a radius vector of $Z_\phi = 2\pi f a_p \sin i/c$, the orbital phase subtended by this tile can be given by:

$$\delta\Phi_{Z_\phi} \approx \frac{\delta Z_\phi}{Z_\phi} = \frac{\sqrt{2\sqrt{3}/5} c}{2\pi f a_p \sin i} \approx \frac{0.86c}{2\pi f a_p \sin i} \quad (radians) , \quad (18)$$

where the 0.86 factor on the right hand side approximates the geometric mean (0.832) between what are, in turn, approximations for the angle subtended by the hexagonal tile flat-flat and diagonal directions (to within 3.3% – the actual mean phase angle subtended by the tiles to the origin of the Z_ϕ plane *is* bigger than what the middle of equation (18) would predict). We can now compare *half* the fineness of this stepping in orbital phase to the worst case mismatch increments (10% loss in power) given by equation (11):

$$\alpha \equiv \frac{\delta\Phi(\Phi = \frac{\pi}{2}, \frac{3\pi}{2})}{0.5 \times \delta\Phi_{Z_\phi}} = \left(\frac{P}{T}\right)^2 , \quad (19)$$

where the approximation in equation (18) has conveniently set the coefficient on the right hand side of equation (19) to unity. Equation (19) gives us the relaxation factor needed for the tile sizes to make the transition from MSP searches in short, to those in longer period binaries. We can now treat the entire search with the short period binary formalism, by letting the tiles expand by a factor of $\max(\alpha, 1)$, as α increases with increasing orbital

period. It is no accident that $2\pi/\sqrt{42} \approx 1 \approx (2\sqrt{3}/5)^{0.5}$, i.e., the larger tile size cuts on just when increasing orbital period, P , or decreasing observation time, T , causes the increments in equation (12), relative to a_p , to exceed those of equation (11).

The step sizes for the orbital period must be small enough so that the drift in orbital phase caused by the period mismatch is less than or equal to the phase subtended by a tile edge (drawn through the tile center) on the Z_ϕ plane:

$$\delta\left(\frac{1}{P}\right)T \equiv \delta FT \approx \frac{0.43c}{4\pi^2 f a_p \sin i} \quad (\text{cycles}) ; \text{ or } : \quad \delta\Omega \approx \frac{0.43c}{2\pi T f a_p \sin i} \quad (\text{radians } s^{-1}) . \quad (20)$$

A point which starts at an offset of x , of a tile edge unit, along the tile diagonal perpendicular to the (radial) direction to the origin of the Z_ϕ plane, and then *moves* by y , of a tile edge unit, in the same direction due to orbital period mismatch (approximating the circumferential arc by a straight line) could easily share/duplicate its Fourier power among *three* different tiles. Assuming losses to Fourier power add directly (as in a quadrature sum) we can compute the total loss of power in the tile of the point's origin, in units of the 10% worst case mismatch, as:

$$\frac{\Delta(P_F)_I}{(P_F)_I} = x^2 + xy + \frac{y^2}{3} . \quad (21)$$

The loss of power to the two tiles whose common edge lies in the path of the point is:

$$\frac{\Delta(P_F)_{II}}{(P_F)_{II}} = 3/4 + (x - 3/2)^2 + (x - 3/2)y + \frac{y^2}{3} . \quad (22)$$

These become equal on the line in the x-y plane given by:

$$y = 2(1 - x) , \quad (23)$$

with a $13/12 \times 10\% = 10.8\%$ maximum loss of Fourier power occurring for the $y = 1$ (and $x = 1/2$) case posited by equation (17). For points in the Z_ϕ plane which begin halfway (radially) between the tile center and edge, proceeding parallel (and thus \sim circumferentially) to those already discussed, the total loss of power in the tile of origin is:

$$\frac{\Delta(P_F)_{III}}{(P_F)_{III}} = 3/16 + x^2 + xy + \frac{y^2}{3} ; \quad (24)$$

and the loss in the downstream neighboring tile is given by:

$$\frac{\Delta(P_F)_{IV}}{(P_F)_{IV}} = 3/16 + (x - 3/2)^2 + xy + \frac{y^2}{3} . \quad (25)$$

These become equal on the line in the x-y plane given by:

$$y = 3/2 - 2x , \quad (26)$$

with a $5/6 \times 10\% = 8.3\%$ maximum loss of Fourier power occurring when $y = 1$ (and $x = 1/4$).

With these steps, we note that the total amount of computation needed by such a search is proportional to the fourth power of f and T , two powers to cover the Z_ϕ plane, another power in the step sizes for the orbital period, and the last factor due to the size of each Fourier transform. Larger worst-case losses will be incurred, of course, by “inter-binning” the resulting Fourier transforms, as much as 19% (even as in MDK) unless more than two Fourier elements are used. At this point, a more sophisticated inter-binning is probably worth the extra computation to reduce the loss to 10% (four elements), or even 6% (six elements). However, the impact of cruder interbinning can be eliminated in two stage post-Fourier analyses, with candidates overselected in the first stage, prior to a second stage that accurately finds the frequencies of their true maximum power, and then uses these as the basis for a search for related harmonics. As the loss of Fourier power produced by the second harmonic ($2f$) structure in the MSP’s pulse profile is *quadruple* that for the fundamental, it is worth including subharmonics in the search algorithm (see §2.4).

When the boundary between short and long periods is reached ($P \geq T$), it is understood that the tile size merely scales by $(P/T)^2$. If the same number of cases are done, the area being covered on the Z_ϕ plane increases. Thus, in almost all practical searches, the *differential* number of cases needed for ($P \geq T$) decreases by $(P/T)^2$.

2.3. Example

As a hypothetical test case, we assume that our target pulsar has a period of 5 ms, i.e., is 4.54 times as fast as the J0737A spin rate of 44.05 Hz. In order to calculate the work necessary to find such a pulsar in a circular orbit with the same period and (since the inclination of the J0737 system is nearly 90 degrees) semimajor axis, we must first calculate the area of the disk on the Z_ϕ plane we need to search. The radius vector of this disk is given by:

$$\rho_m = \frac{2\pi f a_p \sin i}{c} = 1778 \text{ (radians)}, \quad (27)$$

where we distinguish ρ as the radius of the disk on the Z_ϕ plane, where Z_ϕ itself is just the amplitude of the phase modulation as given in equation (14). The number of hexagonal *tiles* in each ring can be counted as *two* for every $3/\sqrt{5}$ radians (when counting the number of *rings* later, we will count these as only *one* ring), with every other tile offset by $1/2$ the flat-flat distance ($\sqrt{3/5} = \sqrt{0.6} \approx 0.775$) in the radial direction, and are separated by $3/(2\sqrt{5})$

radians circumferentially:

$$N_t(\rho) = \frac{2\pi\rho}{3/(2\sqrt{5})} = \frac{4\sqrt{5}}{3}\pi\rho = 16,655 \left(\frac{\rho}{\rho_m}\right) \text{ (tiles)}. \quad (28)$$

Dividing ρ by the flat-flat distance, we can count the number of rings:

$$N_r(\rho) = \frac{\sqrt{15}}{3}\rho = 2,296 \left(\frac{\rho}{\rho_m}\right) \text{ (rings)}. \quad (29)$$

The total number of tiles in the disk of radius, ρ , can then be estimated by the product of the two, times 1/2, due to the integral over the linear ramp of the number of tiles per ring:

$$N_{disk}(\rho) = \frac{1}{2}N_t(\rho) \times N_r(\rho) = 1/2 \frac{4\sqrt{5}}{3}\pi\rho \frac{\sqrt{15}}{3}\rho = \frac{10}{9}\sqrt{3}\pi\rho^2. \quad (30)$$

It would have been easier to just take the disk area, $\pi\rho^2$, and divide by the area per tile, $0.3\sqrt{3}$ radians²:

$$N_{disk}(\rho) = \frac{\pi\rho^2}{0.3\sqrt{3}} = \frac{10}{9}\sqrt{3}\pi\rho^2 = 1.91 \times 10^7 \left(\frac{\rho}{\rho_m}\right)^2. \quad (31)$$

However, to compute the total amount of processing needed we need $N_t(\rho)$ to weight by $N_P(\rho)$, the number of trial orbital periods, prior to integrating (see just below).

Assuming our observation time, T , is 17,669 s, twice the J0737 orbital period of 8,834.5 s (P_7), we will search over a range in orbital frequency, from:

$$\Omega_n = \frac{2\pi}{T} = \frac{\pi}{P_7} = 3.56 \times 10^{-4} \text{ (radians } s^{-1}\text{)}, \quad (32)$$

to:

$$\Omega_x = \frac{2\pi}{P_7} = 7.11 \times 10^{-4} \text{ (radians } s^{-1}\text{)}. \quad (33)$$

For observation times longer than $2P_7 = 17,669$ s, the relaxation factor from equation (19) applies and the amount of added computation, relative to that needed for $2P_7$ to P_7 , is down by a factor of 3. As in equations (20) and (27) the spacing of the orbital frequency search gets finer as ρ increases, so that the drift of the vector on the Z_ϕ plane does not exceed $1/\sqrt{5}$ radians:

$$\delta\Omega = \frac{1}{(\sqrt{5}T\rho)}. \quad (34)$$

When we calculate diminishing trial periods from T to P_7 , the number of trial orbital periods needed in a given ring is:

$$N_P(\rho) = \frac{(\Omega_x - \Omega_n)}{\delta\Omega} = 2\pi\left(\frac{1}{P_7} - \frac{1}{2P_7}\right)\sqrt{5}T\rho = \sqrt{5}\pi\rho = 2.5 \times 10^4 \left(\frac{\rho}{\rho_m}\right) \text{ (trial periods)}. \quad (35)$$

Now we can multiply the number of trial periods per tile, by the number of tiles per ring, and finally by the number of rings to get the total work. This time we need a factor of $1/3$ for the integration of the ρ^2 term:

$$N_{FFT}(\rho) = \frac{1}{3}N_P(\rho)N_t(\rho)N_r(\rho) = \frac{40}{27}\sqrt{15}\pi^2\frac{T}{P_{min}}\rho^3 = 3.18 \times 10^{11}\left(\frac{\rho}{\rho_m}\right)^3. \quad (36)$$

Equation (36) shows why it is much easier to detect the slower component of double pulsar systems, such as the 0.36 Hz J0737B, thus gaining exact knowledge of the orbital period, phase (and eccentricity), and approximate knowledge of the companion’s semi-major axis.

2.4. Mitigating Factors and Other Details

In the example above the typical size for each FFT would be set by an ~ 2 kHz sampling rate, times 8834 s, or about $N = 2^{24}$ samples. Processor speeds are such that each these can be done in under a second. The problem arises, however, in getting the 300 billion processor-seconds. There is usually enough memory local to modern processors to accommodate the relatively short FFT’s, so that the computing can proceed in the “embarrassingly parallel” mode. The large number of trials also requires an extra 25.8 units of power to amortize, which is enough to be significant in just one trial. However, the coherent addition of higher and/or lower harmonics (e.g., as in M00, equation [1]) can make up for this quickly, particularly for the first three harmonics of spin periods of 15 ms or longer. Targeted searches which first find the 0.1% to 0.5% most significant “seed” events in the Fourier transform, can be used to coherently investigate trains of whole, half, and $1/3$ harmonics. These can be evaluated in only one extra pass through the Fourier spectrum, with very little loss by using an interpolation involving at least six complex amplitudes.

Such a technique can avoid missing significant periodic signals due to the statistical fluctuations in the seeds, generally with an expected mean power level between 10 ($-3 + 4.5$) and 17 ($-4.24 + 6$) (i.e., with otherwise a much greater potential loss of power than just 10%) because down fluctuations in all *three* seeds are unlikely, and the coherent summing process, once started, is insensitive to them. In addition, the more extensive searches will, of necessity, have seeds with more power, and hence smaller frequency errors, making them more efficient in the harmonic summation process, particularly when including the whole integer higher harmonics (and thus whole integer multiples of the *error* in the fundamental frequency). Still, this amount of computation is prohibitive.

It *is* possible that by convolving the original Fourier spectrum with perfectly matched filters for the phase modulation between one tile and its neighbors (a process that utilizes

short FFT’s in memory also local to the processors [REM02]) that the majority of the longer FFT’s do not have to be computed. Only six convolution templates need to be used, each representing a residual phase modulation for a vector, with a length of a tile flat-flat ($\sqrt{3}$ tile edges) on the Z_Φ plane. This set of seven tiles forms one “7tile.” Next, we can use six more templates, for vector offsets of $\sqrt{21}$ tile edges (e.g., $[4.5, \sqrt{3}/2]$ etc.) to go from the original tile to the center tiles of the six neighboring 7tiles, and then we can flesh out these neighboring 7tiles by again using the six tile diagonal length convolutions, to form a “49tile.” The steps in orbital period have to as fine as needed for the most radially distant tile in the 49tile. Since we are assuming that even the large FFT’s fit entirely within memory local to each processor and FFT computation in such circumstances takes on the order of $\log_2(N)$ this technique can only reduce the amount of computation required by a factor of four at best, and even then only with a much faster, and likely less effective, harmonic searching algorithm.

3. Discussion

Fortunately, there may be reasons to believe that the MSPs found in DNSBs will, *no matter how the MSPs entered into the pre-DNSB*, have periods longer than 15 milliseconds, even though shorter period MSP members would not be easily detected. Although the predominant species of neutron stars (NSs) are the merger-spawned, weakly magnetized, MSPs with an initial spin period of ~ 2 ms (M04a) at first glance it would seem to be very difficult to incorporate these into DNSBs as the MSP companion. There is not enough time for massive stars to capture them before they suffer their Fe catastrophe SNe, but if this does happen, then a 2 ms MSP in a DNSB becomes a real possibility. The less massive companions can not evolve into pre merger-target white dwarfs, again because the Fe catastrophe of the massive star happens too quickly.

However, it *is* perfectly feasible for accretion to slow an NS rotation that is *retrograde* wrt the orbital angular momentum, a frequent outcome of NS capture as well as post-capture merger-induced core collapse, both of which would most likely result from binary-binary collisions. Thus, if an NS *is* captured, then accretion from the pre-Fe catastrophe massive star could *slow* its spin to 50 Hz, or below. *Double* merger, and/or single merger and a disruption accretion disk, both resulting from binary-binary collisions, could also deliver a weakly magnetized MSP *and* a post merger companion sufficiently massive to progress to an Fe catastrophe SN, thus “speeding up” its evolutionary clock. Whether these mechanisms are prevalent enough to produce the current distribution of DNSBs remains in question. However, condensed or collapsed core (CC) globular cluster (GC) stellar densities don’t

seem to be required for the Galactic plane to make its share of fast ($P \leq 15$ ms) i.e., merger-spawned MSPs, and the numbers of such MSPs in the two categories are about the same. However, given that the GCs are generally farther away (save for B1913+16 and J1811-1736 at 7 and 6 kpc, respectively, closer than 47 Tuc and M4, at 4 kpc and 2 kpc) and on account of their known dispersion measures, their MSPs have been discovered in *longer* time series observations. This selects *against* discovering GC, relative to Galactic plane (GP) DNSBs, thus the current GC/GP DNSB ratio of 1 to 7 is not unexpected for the same formation mechanism in both locations.

Some even hold that accretion onto a *directly* rotating (prograde) NS can also slow its spin rate (Kundt 2002), and there may be some evidence for this in the MSP population of the GCs (see below). In this case, the DNSB *can* gain its MSP through, again most likely, binary-binary collisions in which the MSP companion is captured (but not likely, except in the case of double merger, *formed* by white dwarf-white dwarf merger and subsequent core collapse). An epoch of accretion ensues which then, relatively quickly, *slows* the NS spin. Later, the companion undergoes an Fe catastrophe SN and forms the highly magnetized NS. This opens up the possibility that DNSBs could have MSP components spinning faster than ~ 50 Hz, and the reason none have been found so far would have to be due to the kinds of selection effects discussed here. It is also possible that, because the accretion onto the J0737As must *always* happen if the progenitors of the J0737Bs are to evolve into Fe catastrophe SNe, the 2 ms periods may never occur for the J0737As by the time of the Fe catastrophe SNe of the pre-J0737Bs.

Before getting more specific about the details of the evolution of the J0737 binary system, we can still draw a few preliminary conclusions. Observations prohibit the MSP companions from being produced *directly* by their Fe catastrophe SNe, unless they are born so weakly magnetized as to persist as radio live pulsars beyond the epochs of their companions' impending Fe catastrophe SNe. Although some Fe catastrophe SNe have been born spinning faster than 100 Hz (Marshall et al. 1998), and some 3D calculations have indicated that a weakly magnetized NS could result from one (Fryer & Warren 2004) the very state of the remnants of the last SNe in DNSBs testifies otherwise. Out of eight DNSBs, seven have radio-dead companions, and the last (J0737B) has a 1.2×10^{12} G field and is spinning so slowly (2.8 s) as to have one foot already in the grave. In addition, the numbers of weakly magnetized Fe catastrophe SNe are limited because the gamma ray burst (GRB) rate sets a strict *lower* limit to the incidence of merger-induced SNe (Schmidt 1999, M04a). (In case there remain any lingering doubts that SN 1987A, merger-induced core collapse SNe, and GRBs are intimately related, we note, from M04a, that the product of the energy of the “mystery spot,” 10^{49} ergs, and the beaming factor for GRBs, 10^5 , yields 10^{54} ergs, a fair fraction of the maximum assumed isotropic fluence of GRBs.) Thus, unless we argue, that

in spite of the limited rate, some of the seven are similar to Cas A (Tananbaum 1999), with no evidence of a magnetic field at all, Fe catastrophe SNe can only *rarely* produce NS remnants with the weak fields typical of MSPs, if ever. Field *growth* in pulsars due to magnetic/thermal interaction (Blandford, Applegate, & Hernquist 1983) even after hundreds of years, must also be considered unlikely, again because of Cas A. Thus the eight DNSBs constitute the best evidence so far that Fe catastrophe SNe *usually* produce remnants with magnetic fields well in excess of 10^{10} Gauss.

In addition, PSR J0737A has a spin rate of 44 Hz, and, extrapolating backward, its initial rate, just prior to the pre-J0737B Fe catastrophe SN, was not much higher. This spin rate is more typical of those few MSPs with slower spins (Chen, Middleditch, & Ruderman 1993, hereafter CMR93) and distinctively sharp pulse profiles such as M15 A, B, C, and G (Anderson 1992; Chen & Ruderman 1993, hereafter CR93) and another handful in the other CC GC clusters and Galactic plane, that most agree have actually been recycled. Thus, J0737A must have been accreting from a star massive enough to undergo Fe catastrophe core collapse, and in spite of that was still spinning at only 50 Hz at that epoch some 50 Myr ago. Taken at face value, this supports the picture of predominant, fast (2 ms) merger-induced MSPs, versus the slower, recycled (and hence rare) MSPs.

The “standard” model for DNSB formation starts with two massive, (but not equally so) stars, and requires two different types of accretion at two different epochs. The first involves the evolution of the more massive companion star A (the precursor to J0737A) and unstable mass transfer onto star B (the precursor of J0737B) as a way of avoiding a common envelope inspiral in short period binaries (Battacharya & van den Heuvel 1991; Dewi & van den Heuvel 2004; Willems & Kalogera 2004). Star A then undergoes an Fe catastrophe SN, leaving J0737A with the helium-rich companion, star B, which then evolves and overflows its Roche lobe, beginning the second epoch of mass transfer (this time stable) onto J0737A as a high mass X-ray binary system (HMXB). This era of accretion spins up J0737A to an intermediate rotational frequency, while reducing its effective magnetic dipole moment. Star B evolves and undergoes an Fe catastrophe SN, leaving the highly magnetized NS, J0737B. Following this logic, *if* accretion were the *only* way to get a rapidly spinning NS, then the DNSBs would constitute the best evidence so far that the magnetic field configuration of an NS can be modified by accretion. But, of course, it isn’t (M04a).

The assumption implicit in the standard model, is that a longer interval of accretion in a binary ($\sim 10^9$ yrs) can typically spin up an NS to rotate as fast as 500 Hz, so that a shorter interval of accretion ($\sim 5 \times 10^7$ yrs) while star B evolves toward its Fe catastrophe SN, is sufficient to spin it up from 0 to 50 Hz, and reduce its magnetic field to a value typical of MSPs. However, as already mentioned, GRBs require a certain number of merger-spawned

MSPs born with periods near 2 ms, leaving less and less room in the MSP population for those expected to be recycled into spinning as fast as 500 Hz. In addition, the details of the MSP population in the globulars do not support recycling as the origin of the fast MSPs (2-15 ms, M04a). So perhaps recycling may so ineffective that it could even have a hard time just spinning up J0737A from 0 to 50 Hz in only 5×10^7 yrs.

The similarity of the distribution of MSP spin periods in the DNSBs (23 to 104 ms) to that of the “truly” recycled MSPs in the CC GCs (23 to 111 ms, with the 30.5 ms M15C in common) is uncanny, and, if not due to selection effects, could have physical meaning. It may only go so far, however, as J1811-1736 and B1913+16, with the longest periods of the eight DNSBs (104 and 59 ms) have the broadest (even relative) pulse profiles, $W50(\text{ms})/P0(\text{s}) = 225$ and 207 (ATNF 2004 – note that W50 is a function of the frequency band and time resolution of the observation, so caution must be used), with the rest scattering between 18 and 106. The same relation for the MSPs M15A-H, respectively, yields, in (W50/P0, P0) pairs: (18,111), (61,56), (43,31), (271,4.8), (172,4.65), (197,4.0), (21,38), and (104,6.7) (Anderson 1992) and thus shows the opposite trend. The relation in DNSBs may be due to varying amounts of accretion, while that in M15 may be due mostly to age. However, age may also have affected B1913+16, in that it has slowed significantly in just the last 10^8 years, leaving the putative varying spinup-caused direct relation between P0 and W50/P0 intact only for J1811-1736. If we ignore J1811-1736, then M15C and J0737, with periods of 30.5 and 22.7 ms, have W50/P0s of 43 and 106, consistent with the trend in the CC GCs, but the evidence is sparse, *and* M15C belongs to both distributions, so the issue of similarity in the trend of W50/P0 between the DNS, and the CC GC MSPs remains undecided.

The W50/P0 versus P0 relationship for the *entire* pulsar population has a power law trend with an index near -0.8, indicating wider pulse profiles with decreasing period, until it reaches the (mostly recycled) MSPs at $P0 = 0.1$ s, at which point there are many pulsars with pulse profiles that are too narrow, such as M15A, C, and G. These start a new trend with roughly the same power law, that connects to the MSPs with periods shorter than 23 ms. There is a gap here, but this is due to not having W50s for the 28.8 ms pulsar, J0621+1002, and the 23.1 ms pulsar, J1804-0735 in the GC NGC 6539, and, of course, J0737A will also help fill it. The new trend is about a factor of ten below (toward narrower pulse profiles) any W50/P0 value of the extrapolation from the longest period pulsars, which is good because the continuation of the upper track exceeds 1000 near $P0 = 10$ ms, thus becoming unphysical (the ~ 1.6 ms pulsars B1937+21 and B1957+20 fall below even the *lower* track). The dichotomy of the slow/narrow versus the fast/broad pulsars in M15 and the other CC GCs then, can also be viewed as consequence of this trend, and the near order of magnitude geometric mean difference factor between the sets of long and short spin periods. So instead of asking why some pulse profiles are wider than others, it may make

more sense to ask why the distribution of spin *periods* of the CC GCs is bimodal.

All of this begs the question of if and how the recycling history of seven of eight DNSB MSPs differs from that of the CC GC MSPs, and how is the configuration of the NS magnetic field affected by each and why? The recycled GC MSPs, including those in M15, certainly had no several-solar mass helium star from which to accrete, so they must have managed with leaner pickings, or, again, the accretion itself slowed them. What is apparent from “normal” accretion is that, the more magnetized the NS, the slower the equilibrium spin frequency, from MSPs (though this evidence is sparse), through the HMXBs, and on down to AXPs/magnetars. Somehow, the field manages to get modified and we’re still unsure if we have ever witnessed this process. Thus, if only *non-braking* accretion occurs, there obviously have to be a few different states, from “super-critical” on down, that have different effects on NS magnetic field configuration, and it is clear that particular extreme is not happening in HMXBs such as Cen X-3 and SMC X-1, or even Her X-1.

Both the standout ~ 1.6 ms pulsars, B1937+21 (Backer et al. 1982) and B1957+20 (Fruchter, Stinebring, & Taylor 1988), have very narrow (double) pulse profiles consistent with magnetic to (single) rotational pole migration due to accretion stresses during recycling, without their (now very close together) polar surface field *strengths* being significantly attenuated (CR93). As mentioned above, these are both located at the bottom extreme of the high frequency end of of the lower W50/P0 vs P0 track. *If* accretion *can* slow the spin of most NSs, then it is tempting to say that *these two* quite possibly form the *entire* class of (initially) strongly magnetized, and later, recycled MSPs, with *all* of the rest having weak magnetic fields at birth, through recycling (or not), and afterward. In any case, they are the rarest of rare pulsars, being first selected against, by as much as a factor of a hundred, because they were offspring of Fe catastrophe SNe (M04a) and then (perhaps) again by another factor of 10, because they somehow had to have very special accretion/field configuration-modifying circumstances. The number of “standard” MSPs exceeds that of the “non-standard” MSPs, B1937+21 and B1957+20, by about the same factor that the number of merger-induced SNe exceeds the number of Fe catastrophe-induced SNe, lending support to the possibility that accretion can act differently on the fewer strongly magnetized NSs than it does on their much more numerous, but *less* magnetized cousins. Still we do not yet know *how* accretion spins up a few strongly magnetized NSs, such as B1937+21 and B1957+20, to very high rotation rates, and why it is *not* all that effective at doing the same with the weakly magnetized (per Alpar et al. 1982), but pulsar recycling appears to work this way. Studies of this problem are still in their infancy (Cumming 2004). For now, however, we must conclude that there is a remote possibility that a 1.6 ms pulsar will be a member of a DNSB.

The last remaining holdout is J0034-0534, a pulsar in a 1.6 day binary with a low mass companion in a circular orbit of ~ 10 lt-s (Bailes et al. 1994). This unique pulsar has a record low magnetic field and near record broad pulse profile, a 1.87 ms spin period, and a very steep radio spectrum (ATNF 2004), all of which make it a candidate for a merger of a white dwarf with a weakly magnetized NS (Middleditch 2004b). The spin period is shorter (just) than the 2.1 ms minimum period for merger-spawned pulsars (Camilo et al. 2000; M00) and longer than ~ 1.6 ms (the “Chen Gap,” after Kaiyou Chen). The pulse profile, however, is not dramatically wider in W50/P0 than the mean of the low P0 distribution extrapolated to this frequency¹ but, at this point there is very little *room* for increase. Again, there is only a very remote possibility that such a pulsar will be a member of a DNSB.

The more likely cousins to J0737, and certainly the easiest to find, short of detecting the younger, slower, strongly magnetized member, may be other similar systems, with a modestly fast MSP, like J0737A, and shorter or longer orbital periods, but at this point, we can not rule out selection effects favoring slower MSPs over faster. Nevertheless, we take, as an example, a pulsar with the same total mass of 2.58 solar, in a binary system with an orbital period of 1472 s, six times shorter than P_7 , the 8,834 s orbital period of J0737, *and* our assumed observation interval. To keep the argument simple, we relax the constraint on total mass for orbital periods longer than 1,472 s. Otherwise, ρ_m scales as $(P/(1,472s))^{2/3}$ and the work scales as $(P/(1,472s))^2$ as P gets larger. For this case, $a_p \sin i = 0.43$ lt-s and $\rho_m = 119$ radians. The amount of computation needed for orbital periods longer than P_7 , relative to that required for P_7 to $P_7/6$, is down by a factor of 15. The number of trial orbital periods needed at ρ_m for the range of 8,834 s to 1472 s is $N_P = 8,332$. The number of tiles/ring at ρ_m , $N_t = 1,111$. And finally, the number of rings at ρ_m , $N_r = 153$. In this case the amount of computation needed at ρ_m (recall the factor of 1/3 in equation [36]) is only 4.73×10^8 , nearly a factor of a thousand less, and very accessible to modern computers.

The reduction is due to two factors, the first being the decrease by a factor of 3.3 of the semimajor axis, which produces a factor of 36 when cubed. The second is the factor of 4.54 reduction in pulsar frequency, from 200 to 44 Hz, producing a factor of 94. Together these produce a factor of 3,375, or the cube root (15) in ρ_m (*unrelated* to the factor of 15 which applies to the long-to-infinite orbital period cases posited above). Against this, we used more trial orbital periods by a factor of 5 in the 1,472 s example, so our net reduction in calculation is about 675. The Fourier transforms are also shorter for this example, but the bad news is that the associated harmonic searches will work well only for 22 Hz and lower spin frequencies (for the 2nd harmonic), and 14 Hz and slower for the third harmonic. The good news is that

¹Ignoring the erroneous values of 998 for J2051-0827 (the true value is about 60 – Stappers et al. 1996) and the additionally *unreferenced* value of ~ 3600 for J1807-2459!

the work required for fixed mass scales as P^2 as P gets smaller, while, at the same time, the pulsar orbit is more likely to be circular. Other alternatives include just reducing the observation time analyzed by a factor of 4, to 2,208 s, which would sacrifice a factor of 2 in signal-to-noise ratio, but which would also change the fully computationally intensive upper limit period from 8,834 s down to 2,208 s, reducing the amount of computation to 2,208 s by a nearly a factor near 2.3, in addition to the factor of 256 reduction everywhere, counting FFT lengths. Like the original example in §2.3 such a search would be effective up to the third harmonic of a 15 ms pulsar.

Of course, if we want to search for MSPs with black hole companions, the total mass of the binary system will almost certainly exceed three solar, with the required amount of computation rising steeply with black hole mass.

4. Conclusion

In this work we have derived the form of the transition between acceleration binary pulsar searches and truly short period binary pulsar searches. We have also shown here that although the amount of computation needed to search for MSPs in short period-high companion mass binary systems appears to be stupendous, rising as the fourth power of both the observation time and targeted upper limit pulse frequency, many such searches are now within our grasp, given intelligent choices of ranges for the unknown parameters. Recent advances in the understanding of the nature of supernovae and their compact remnants (e.g., M00, M04a, and this work, §3) can play a crucial role in narrowing otherwise prohibitively expensive computer searches for MSPs in binary systems. The easiest, and perhaps, most likely candidates for more DNSBs are similar to J0737 (though this is far from certain) with a weakly magnetized NS spinning only moderately fast. The possibility of finding any pulsar in a DNSB, with a ~ 10 to 1.5 ms period, appears to be very remote, but many crucial details of DNSB formation and the pulsar recycling process remain mysteries.

The astonishing similarity between the MSP periods in the DNSBs and the those of the longer period MSPs found only in the CC GCs, including the one MSP/DNSB in common, M15C, calls the standard model, which requires back accretion from a several solar mass, pre-Fe catastrophe SN helium star, into question, in spite of what Lorimer et al. (2004) would have us believe. Other options include MSP formation via binary-binary collisions and *spindown* via the reverse accretion.

For the short period X-ray binary systems in particular, the range of parameters for such searches can be narrowed with exact knowledge of the orbital period and approximate

knowledge of the pulsar candidate’s projected semi-major axis and orbital phase. Searches for MSPs in binary systems where one pulsar is already known, can be narrowed even further, as the orbital period, phase, and eccentricity are known exactly. However, slowly-rotating (non-MSP) pulsar companions in DNSBs do not require a very sophisticated search in order to set stringent upper limits, and eventually Relativity will likely provide *all* the orbital elements for the undetected companion NS, although these have much shorter radio lifetimes. Finally, even though this work does not discuss how to treat eccentricity, it can serve as a guide in the cases for which it is known exactly.

We thank Scott Ransom for discussions. This work was performed under the auspices of the Department of Energy, and supported in part by the Institute for Nuclear and Particle Astrophysics and Cosmology (INPAC).

REFERENCES

- Alpar, M. A., Cheng, A. F., Ruderman, M. A., & Shaham, J. 1982, *Nature*, 300, 728
- Anderson, S. B. 1992, PhD thesis, California Institute of Technology
- ATNF 2004, www.atnf.csiro.au/research/pulsar/psrcat
- Backer, D. C., Kulkarni, S. R., Heiles, C., Davis, M. M., & Goss, W. M. 1982, *Nature*, 300, 615
- Bailes, M., et al. 1994, *ApJ*, 481, 386
- Battacharya, D., & van den Heuvel, E. P. J. 1991, *Phys. Rep.*, 481, 386
- Blandford, R. D., Applegate, J. H., & Hernquist, L. 1983, *MNRAS*, 204, 1025
- Burgay, M., et al. 2003, *Nature*, 426, 531
- Camilo, F., Lorimer, D. R., Freire, P., Lyne, A. G., & Manchester, R. N. 2000, *ApJ*, 535, 975
- Champion, D. J., Lorimer, D. R., McLaughlin, M. A., Cordes, J. M., Arzoumanian, Z., Weisberg, J. M., & Taylor, J. H. 2004, preprint (astro-ph/0403553)
- Chen, K., Middleditch, J., & Ruderman, M. A. 1993, *ApJ*, 195, L51 (CMR93)
- Chen, K., & Ruderman, M. A. 1993, *ApJ*, 408, 179 (CR93)
- Cumming, A. 2004, preprint (astro-ph/0404518)
- Dewi, J. D. M., & van den Heuvel, E. P. J. 2004, *MNRAS*, 349, 169
- Dib, R., Ransom, S., Ray, P., & Kaspi, V. 2004, preprint (astro-ph/0401134)
- Faulkner, A. J., et al. 2004, in preparation
- Fruchter, A. S., Stinebring, D. R., & Taylor, J. H. 1975, *ApJ*, 195, L51
- Fryer, C. L., & Warren, M. S. 2004, *ApJ*, 601, 391
- Hulse, R., & Taylor, J. H. 1975, *ApJ*, 195, L71
- Kalogera, V., et al. 2004, *ApJ*, 601, L179

- Kundt, W. 2002, in Proc of the 270. WE-Hearus Seminar, Neutron Stars, Pulsars and Supernova Remnants, ed. W. Becker, H. Lesch, & J. Trumper, MPE, 278, 206
- Lorimer, D. R., et al. 2004, preprint (astro-ph/0404274)
- Lyne, A. G., et al. 2000, MNRAS, 312, 698
- Lyne, A. G., et al. 2004, Science, 123, 456
- Marshall, F. E., Gotthelf, E. V., Zhang, W., Middleditch, J., & Wang, Q. D. 1998, ApJ, 499, L79
- Middleditch, J. 2004a, ApJ, 601, L167 (M04a)
- Middleditch, J. 2004b, abstract, LAUR 04-2113 (M04b)
- Middleditch J., & Cordova, F. A. 1982, ApJ, 255, 585
- Middleditch, J., Deich, W. T. S. & Kulkarni, S. R. 1992, in Isolated Pulsars, ed K. A. Van Riper, R. Epstein, & C. Ho (Cambridge Univ. Press), 372 (MDK)
- Middleditch, J., et al. 2000, NewA, 5, 243 (M00)
- Middleditch, J., & Nelson, J. E. 1976, ApJ, 208, 567
- Nelson, R. W., Finn, L. S., & Wasserman, I. 1990, ApJ, 346, 226
- Nice, D. J., Sayer, R. W., & Taylor, J. H. 1996, ApJ, 466, L87
- Prince, T. A., Anderson, S. B., Kulkarni, S. R., & Wolszczan, A. 1991, ApJ, 374, L41
- Ransom, S. M., Cordes, J. M., & Eikenberry, S. S. 2003, ApJ, 589, 911
- Ransom, S. M., Eikenberry, S. S., & Middleditch, J. 2002, AJ, 124, 1788
- Schmidt, M. 1999, ApJ, 523, L117
- Stappers, B. W., et al. 1996, ApJ, 465, L119
- Tananbaum, H. 1999, IAU Circ. 7246
- Willems, B., & Kalogera, V. 2004, ApJ, 603, L101
- Wolszczan, A. 1991, Nature 350, 688